Composite Overlapping Meshes for the Solution of Radiation Forces on Submerged-Plate

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The purpose of this study is to predict and understand the hydrodynamic forces and their nonlinear behaviors of fluid motion around the submerged plate oscillating near a free surface. To achieve this objective, we have developed a composite grid method for the solution of a radiation problem. The domain is divided into two different grids ; one is a moving grid system and the other is a fixed grid system. The moving grid is employed for the body fitted coordinate system and moves with the body. This numerical method is applied to calculation of radiation forces generated by the submerged plate oscillating near a free surface. In order to investigate the characteristics of the radiation forces, the forced heaving tests have been performed with several amplitudes and different submergences near a free surface. These experimental results are compared with the numerical ones obtained by the present method and a linear potential theory. As a result, we can confirm the accuracy of the present method. Finally, the effect of nonlinear and viscous damping has been evaluated on the hydrodynamic forces acting on the submerged plate.

Key Words : Composite Grid Method, Submerged Plate, Radiation Forces, Viscous Damping, Free Surface Effect

1. Introduction

There are many studies about the hydroelastic response of a Very Large Floating Structure (VLFS), and some of them have proposed a breakwater to reduce hydroelastic deformation. Takaki et al. (2001) proposed a newly-designed floating breakwater system that could increase the merits of VLFS. The system consists of Floating Breakwater using Submerged Plate (FBSP) and VLFS with attached submerged plate. The submerged plate built into VLFS is called as 'the third submerged plate'. Fujikubo et al. (2002) and Takaki et al. (2002) carried out the studies of effect of the third submerged plate to reduce the hydroelastic response of VLFS. In particular, it has made clear that the third submerged plate can reduce the wave exciting force in short waves (Takaki et al., 2002).

The free surface effect can strongly influence the added mass and damping coefficient values as a function of frequency when the submerged body oscillates near a free surface (Chung, 1977). The flow is highly nonlinear even at very small amplitude of oscillation, when the body is slightly submerged. The purpose of this study is to predict and understand the hydrodynamic forces and their nonlinear behaviors of fluid motion around the submerged plate oscillating near a free surface. To achieve this objective, we have developed a composite grid method for the solution of a radiation problem. In case of the radiation problem, it is difficult to deal with the relative motion between the moving body and free surface. Thus, we divide the domain into two different grids ;

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one is a moving grid system and the other is a fixed grid system. The moving grid system is employed for the body fitted coordinate at the submerged plate, and it is forced to oscillate sinusoidally with the body. The advantages of this approach are that the complex domain is dealt with more easily and it can be used to follow the moving body.

This numerical method is applied to calculation of the radiation forces generated by the submerged plate oscillating near a free surface. In order to investigate the characteristics of the radiation forces, we have performed the forced heaving tests with several amplitudes and different submergences near a free surface. These experimental results are compared with the numerical ones obtained by the present method and a linear potential theory, and we discuss the effect of nonlinear and viscous damping on the hydrodynamic forces acting on the submerged plate.

2. Overlapping Grid System

2.1 Composite grid method

The composite grid method has some attractive features. Firstly, the composite grid method uses a set of independent overlapping grid systems, and the flow information is transferred from one grid to another by an interpolation procedure. Thus, one is able to divide the domain into different blocks and adapt the best type of grid and resolution necessary to each other. Therefore, higher resolutions can be used in some part of the physical domain where it is necessary. Another great advantage of the composite grid method is its ability to handle body motions without regeneration of the grid at each time step as for the single grid system. In such a case, the overlap region changes with time and has to be determined together with the interpolation factors after time step. The only additional effort is the interpolation from one reference frame to the other at the interfaces. Grids of this kind are called Chimera grids in the literature.

The composite grid, which is used in this study, consists of two different grid systems, one is a moving grid system and the other is a fixed grid system. The moving grid system is applied for the submerged plate oscillating near a free surface to deal with the relative motion between the moving plate and free surface, while the fixed grid system covers the surrounding of moving grid and whole of the computational domain. The flow information is transferred from one grid to another by an interpolation procedure. Figure 1 shows the overlapping grid system along with the close up view of the overlapping region.

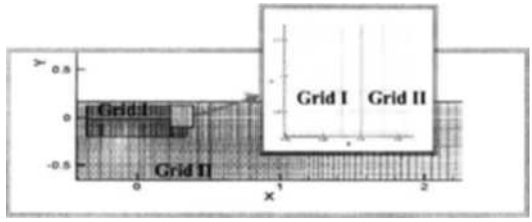


Fig. 1 Overlapping grid system

The pressures are computed simultaneously on the entire flow field until convergence, while the momentum equations are solved independently on each sub-domain. Schwarz had proposed an alternating solution procedure for the elliptic function problems (Hinatsu and Ferziger, 1991). Therefore the Schwarz iteration is used to calculate the pressure equation over the composite grid.

In order to use the Schwarz iteration, we need the interior boundary value which lays in the overlap region and is obtained by interpolating from the other grid.

The algorithms are as follows ;

(1) Calculate the pressure on grid-I using the interior boundary value.

(2) The interior boundary value for grid-II is obtained by interpolation on grid-I.

(3) Calculate the pressure on grid-II using the interior boundary value.

(4) Update the interior boundary value for grid-I using the interpolated data from grid-II.

(5) Repeat (1)-(4) steps until the solution converges.

2.2 Interpolation method

The Newton-Raphson interpolation method is employed at the different grids to transmit the

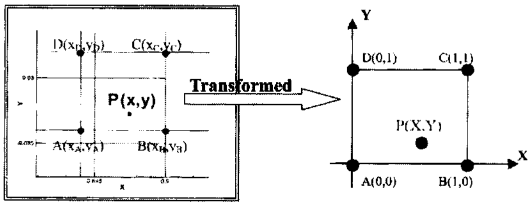


Fig. 2 Transformation of physical coordinate for Newton-Raphson interpolation

flow data from one grid to another.

The overlap regions change with time and flagging of all the grid points is performed after each time step. In the overlap region, boundary conditions for one grid are obtained by interpolating from the other grid.

The interpolated data can be expressed by

$$\phi=(1-X)(1-Y)\phi_A+X(1-Y)\phi_B \\ +XY\phi_C+(1-X)Y\phi_D \quad (1)$$

where ϕ is the interpolated flow data, ϕ_A, ϕ_B, ϕ_C and ϕ_D are the value at the corner A, B, C and D of the cell, respectively. (X, Y) is the local coordinate of the interpolation point in the transformed system obtained by solving the following equations;

$$x=(1-X)(1-Y)x_A+X(1-Y)x_B \\ +XYx_C+(1-X)Yx_D \quad (2)$$

$$y=(1-X)(1-Y)y_A+X(1-Y)y_B \\ +XYy_C+(1-X)Yy_D \quad (3)$$

where (x, y) is the physical coordinate of the interpolated point.

Figure 2 shows the transformation of the physical coordinate to the local coordinate by using the Newton-Raphson interpolation method.

3. Numerical Procedure

The governing equations are the Navier-Stokes equation and the continuity equation for 2-dimensional, incompressible and viscous fluid. They can be written as

$$\nabla\cdot U=0 \quad (4)$$

$$\frac{\partial U}{\partial t}+\nabla\cdot(U-V)U=-\frac{\nabla P}{\rho}+\nu\nabla^2 U-g \quad (5)$$

where ρ is the fluid density, $U=(u, w)$ the fluid velocity vector, V the moving velocity of grid, P the pressure, ∇ the gradient operator, ν the kinematic viscosity, and g is the gravitational acceleration. By comparing with the fixed grid system, velocity of the moving grid is included in the convective terms (Demirdzic and Peric, 1990). The same Eqs. (4) and (5) are used for the fixed and moving grids except that the moving grid velocity V becomes zero on the fixed grid. The computational procedure is similar to the modified $TUMMAC$-V_{WV} method which incorporate the subgrid-scale (SGS) turbulent model (Lee et al., 1990).

The velocity components are advanced explicitly and the pressure is obtained by solving a Poisson equation simultaneously using the Schwarz iterative method on the entire domain. The momentum equations are solved independently on each sub-domain. Interpolation on the overlapping grids is computed by the Newton-Raphson method.

Zero-normal gradient conditions are given for the velocity and pressure at the bottom and outflow boundaries of the computational domain. We assume the axis symmetry condition on the center of entire domain.

The body is forced to heave in the form of

$$z(t)=z_a\sin(\omega t) \quad (6)$$

where z_a is the amplitude of oscillation. On the body surface, no flux and no slip conditions are imposed by

$$u=0;\ w=z_a\omega\cos(\omega t) \quad (7)$$

The body is set into motion from a quiescent state, that is, the velocity and free surface elevation are zero at initial time $t=0$.

4. Application of the Composite Grid Method

The convergence tests of vertical and horizontal cell size to determine the suitable condition of computation for the modified MAC method were carried out by Park et al. (2001). Figure 3 shows the results of numerical convergence tests

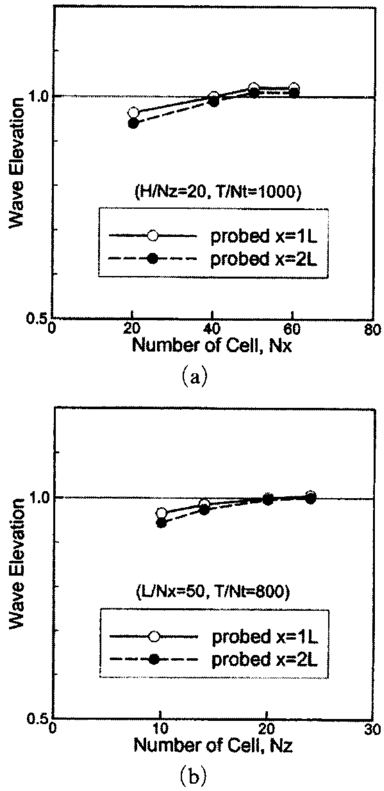


Fig. 3 Results of numerical convergence tests for: (a) Horizontal cell size, and (b) Vertical cell size (Park et al., 2001)

performed by Park et al. for horizontal and vertical cell size, where Nx and Nz indicate the number of cells in a wave length and height. The results seem to converge as the discretized numbers increase. It is evident that the mesh size has to be chosen small to achieve the desirable accuracy.

In order to validate the numerical scheme in this study, the numerical convergence tests are carried out for generation of the regular waves. The wave period, T, is 1sec, the wave length, λ, 1.56 m, and wave height, H, 0.04 m. Two cases of numerical simulation with coarser ($\Delta x=\lambda/40$, $\Delta z=H/10$) and finer ($\Delta x=\lambda/50$, $\Delta z=H/20$) conditions are tested. Figure 4 shows the dependency of grid size through comparison of wave elevation. It is apparently seen that the generation of regular waves are simulated better for finer grid system than for coarser grid. Thus, we

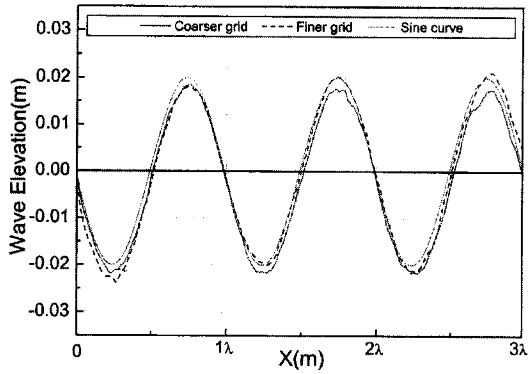


Fig. 4 Comparison of simulated wave elevation for regular waves

adapted this finer grid for the numerical simulation to solve the radiation problem.

4.1 Comparison with the mono-grid system

We have carried out the numerical simulation to compare with the mono-grid system using the rectangular model correspond to length $(L)=100$ mm, submergence $(d)=50$ mm, amplitude $(z_a)=$ 5 mm, and oscillation period $(T)=1.0$ seconds. This computation is to check the Schwarz iteration and interpolation method, thus we do not move the rectangular model and inner grid. Instead, we give the velocity on the body surface for the body boundary condition. Figures 5 and 6 show the pressure contours and velocity vector fields on both composite and mono-grid. Com-

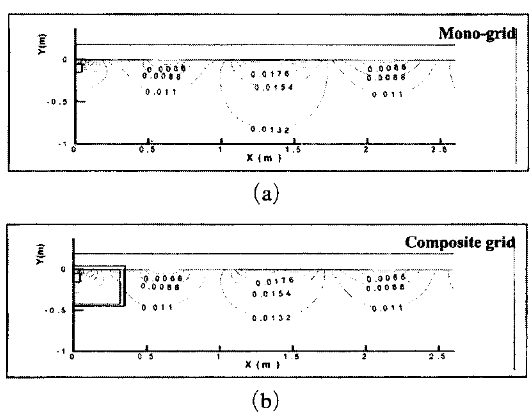
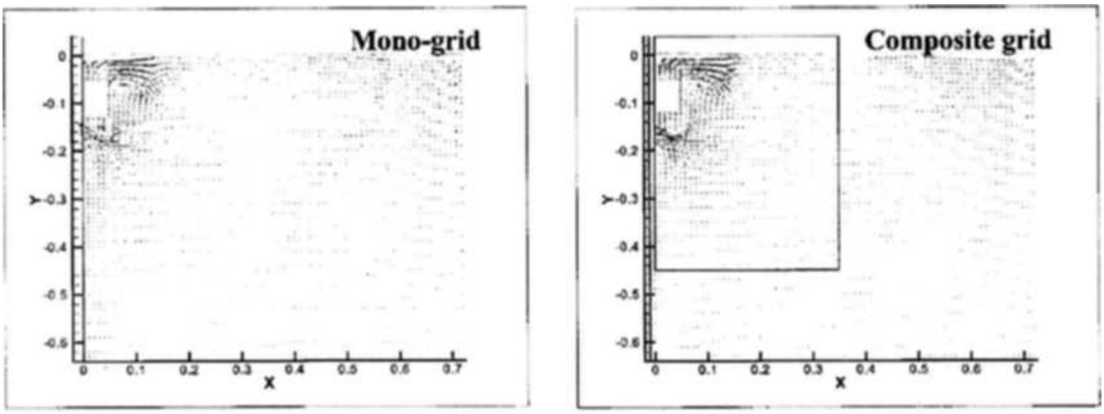


Fig. 5 Pressure contour for $T=1.0$ sec., $d=0.05$ m, $z_a=0.005$ m of (a) Mono-grid and (b) Composite grid

parison of the composite grid with mono-grid shows reasonably good agreement. Therefore we apply the composite grid method to the submerged plate oscillating near a free surface.

4.2 Results and discussion

The forced heaving tests have been performed to investigate the characteristics of hydrodynamic forces on the submerged plate in a two-di-



(a) (b)

Fig. 6 Velocity vector field for $T=1.0$ sec., $d=0.05$ m, $z_a=0.005$ m of (a) Mono-grid and (b) Composite grid

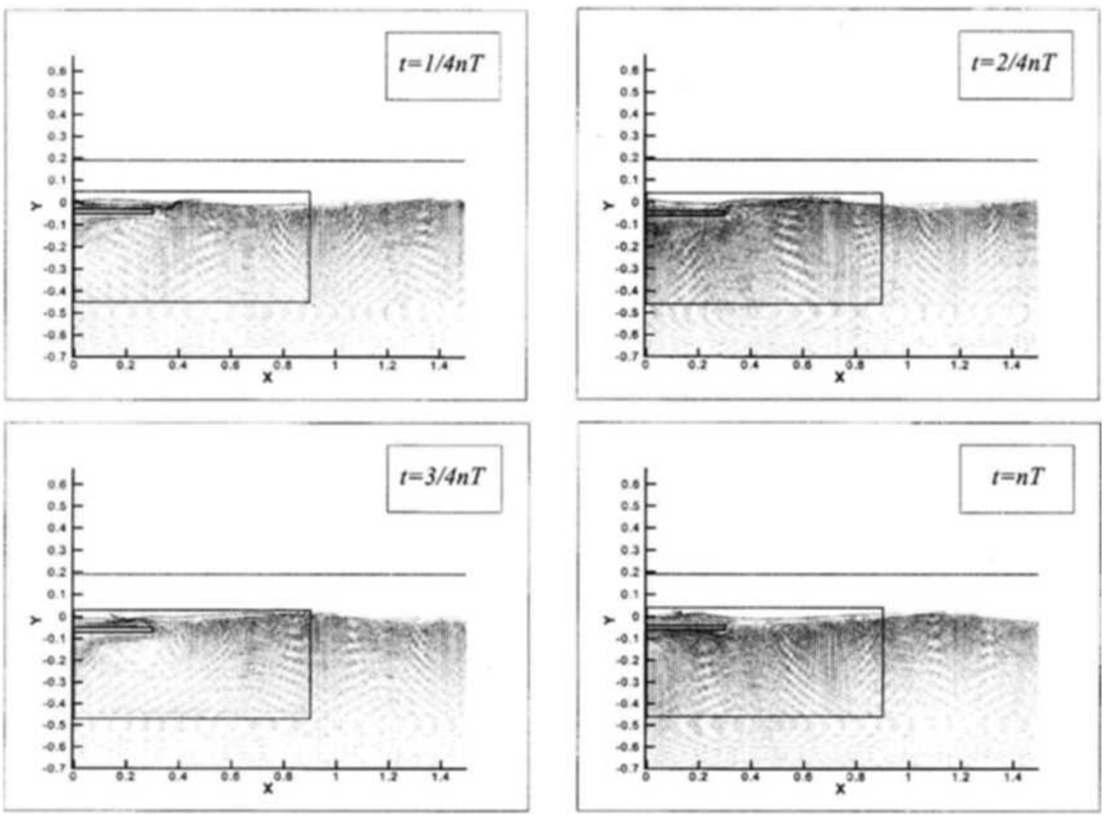


Fig. 7 Time sequences of velocity vector field for $T=0.8$ sec., $d=0.04$ m, $z_a=0.01$ m

mensional wave tank. The height of radiation wave is measured at a distance of 5 m from the center of the model. The forced oscillation experiments have been conducted for the 1/50 scale model under the different test conditions ; d=20, 40 and 60 mm, z_a=10, 20 and 30 mm, T=0.8–2.6 seconds. We also have carried out the numerical simulation based on the composite grid method under the above assumptions.

Figures 7 and 8 show the velocity vector fields of the submerged plate corresponding to the short oscillation period (T=0.8 sec.) and long oscillation period (T=2.2 sec.), d=40 mm, and z_a=10 mm. Here, the inner grid is moving with the submerged plate. It can be observed that the flow incoming from the end of the submerged plate is splashed above the plate. Upon splashing, the surface flow is joined to the outgoing wave. It is shown that the free surface is connected smoothly between the moving grid and fixed grid system, and the radiation waves are properly propagated to outward direction. At the long oscillation period, it can be seen that the vortex ring generated at the edge of the submerged plate is transported to the outward direction underneath the free surface (see Fig. 8). From Figs. 7 and 8, it is clear that the spatial evolution of vortex structure depends on the frequency of oscillation. One can show that the vortical component of hydrodynamic force is related to the moment of vorticity (Lighthill, 1986). Thus, it becomes apparent that the vortical force would depend on the frequency of oscillation. According to this fact, it is considered that the generation of vortex is affecting to the damping forces on the moving body at the long oscillation periods.

The free surface non-linearity and the interaction between wave and body-generated vortical motions could be a significant factor affecting the hydrodynamic forces on the body. Thus, we

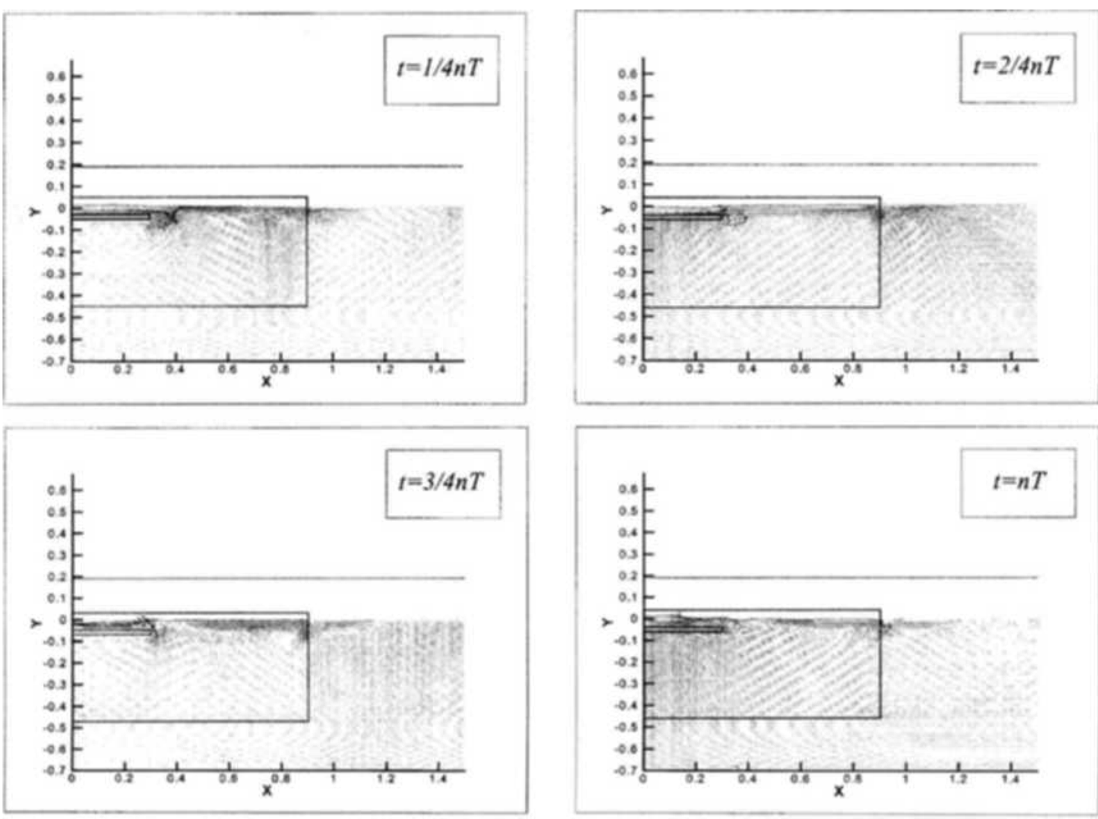
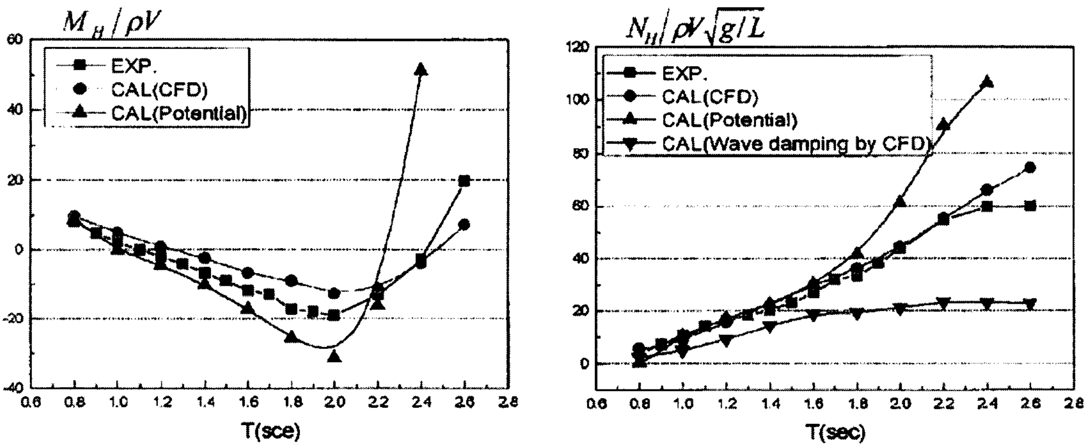


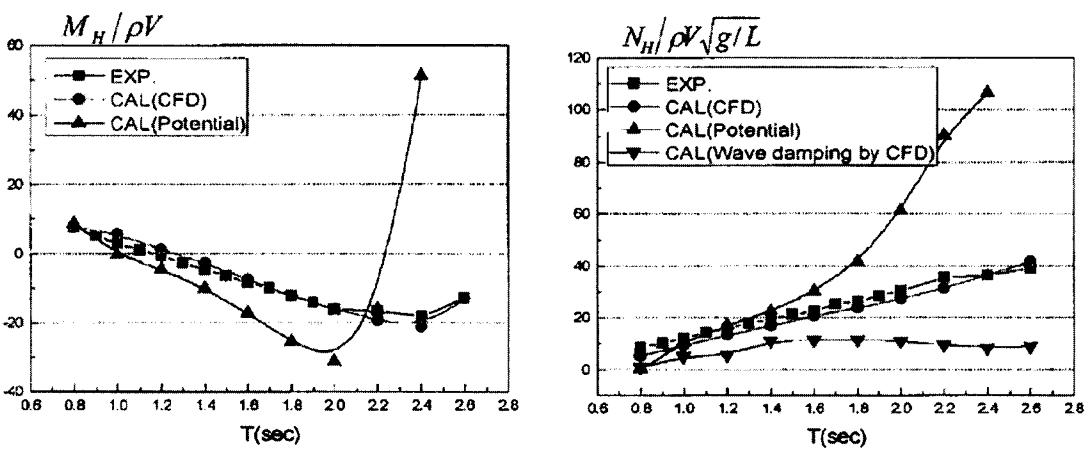
Fig. 8 Time sequences of velocity vector field for T=2.2 sec., d=0.04 m, z_a=0.01 m

have performed the numerical simulation based on the composite grid method for the solution of the radiation problem in viscous fluids. Also, we have computed a singularity distribution method to solve the counterpart of inviscid-flow problem. In order to investigate the effect of viscosity in the radiation hydrodynamic problem, we compare the results of heave force in viscous and inviscid fluids. Figures 9 and 10 show the added mass (M_H) and damping force coefficients (N_H) corresponding to the different amplitude (z_a) = 10 and 30 mm at same submergence depth (d=40 mm). By comparing these results, the results due to the numerical simulation based on the composite grid method agree well with the experimental ones regardless of the amplitude and oscillation periods. On the other hand, the results due to the linear potential theory agree with those of others at the short oscillation periods. However, we notice that the linear theory results considerably deviate from the others at the long oscillation periods. In general, it is noted that the



(a) (b)

Fig. 9 Comparison of (a) Added mass and (b) Dampig force coefficient with experiment for d=0.04 m, z_a=0.01 m

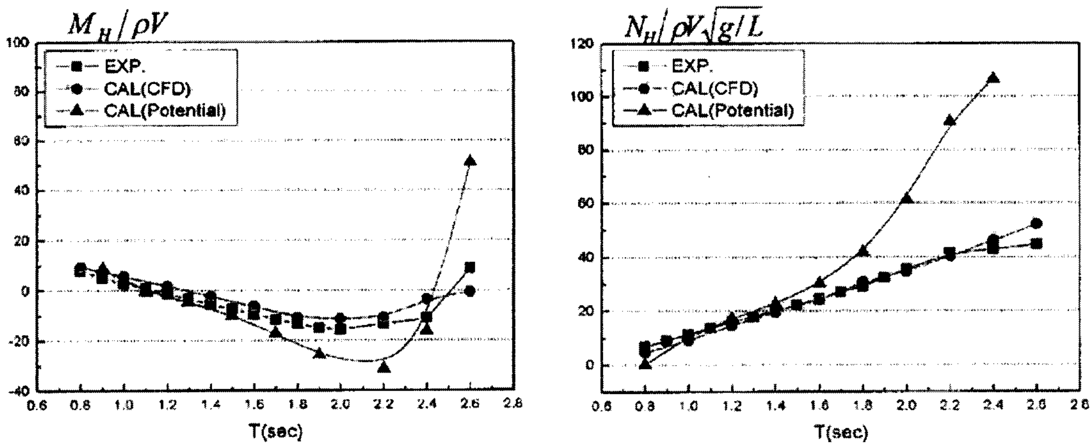


(a) (b)

Fig. 10 Comparison of (a) Added mass and (b) Dampig force coefficient with experiment for d=0.04 m, z_a=0.03 m differences between the linear theory and experiments increase at the large amplitude of oscillation (z_a=30 mm). It can be observed that the components of viscous damping force are larger than those of wave damping force at the long oscillation periods. In summary, these results seem to confirm that the effect of viscosity on the hydrodynamic force is significant at the long oscillation periods.

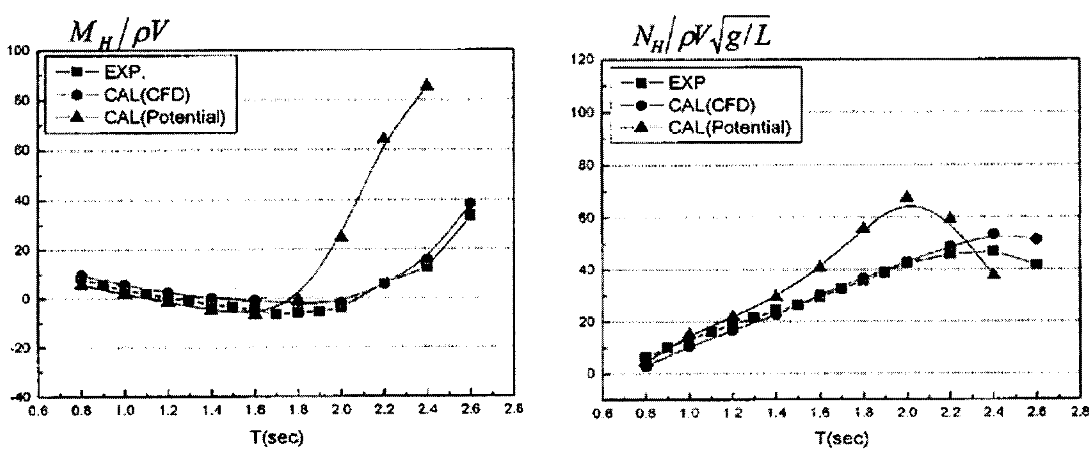
It is noted that the added mass takes negative value for intermediate oscillation periods. Response amplitude of the submerged plate is inversely proportional to both the restoring force and inertia force. Thus the negative added mass means the increase of restoring force. Therefore the submerged plate has the effect of reducing the hydroelastic deformation of VLFS for longer waves.

The added mass (M_H) and damping force coefficients (N_H) are shown in Figs. 11 and 12 for the different submergence depth (d)=40 and 60 mm at same amplitude (z_a=20 mm). From Figs.



(a) (b)

Fig. 11 Comparison of (a) Added mass and (b) Dampig force coefficient with experiment for z_a=0.02 m, d=0.04 m



(a) (b)

Fig. 12 Comparison of (a) Added mass and (b) Dampig force coefficient with experiment for z_a=0.02 m, d=0.06 m

11 and 12, we notice that the deviations of the linear theory results become larger with decrease of submergence depth. This means that the radiation force becomes highly nonlinear at the long oscillation periods due to increase of the viscous damping forces, and the free surface effect increases when the submerged plate come close to the free surface. Also we can observe the negative added mass for the intermediate oscillation periods at the shallow submergence depth (d= 40 mm). On the other hand, the negative added mass become smaller due to decrease of free surface effect at the deep submergence depth (d= 60 mm). Finally, we compare the results of damping forces between the shallow (d=40 mm) and deep (d=60 mm) submergence depth. As fore mentioned, we notice that the effect of viscosity on the hydrodynamic force is rather small at the short oscillation periods but quite significant at the long oscillation periods.

5. Conclusion

In this study, we have developed a numerical method for the hydrodynamic forces on the submerged plate oscillating near a free surface using the composite grid method. Then we have performed the numerical simulation to estimate the radiation forces. The simulation results show good agreement with the experimental ones. This confirms that the present method is reliable and accurate. We observed the occurrence of negative added mass at intermediate oscillation periods, which could have reduced the hydroelastic deformation of VLFS. Finally, we have evaluated the effect of viscosity by comparing with the linear theory. As a result, it is noted that the effect of viscosity on the hydrodynamic forces is significant at the long oscillation periods, especially the deviations between the linear and nonlinear cases are strongly dependent on the amplitude and submergence depth.

Although only heave motion is considered here, this numerical method can be extended to the other motion modes such as surge and pitch, or their combination.

In near future, we will apply this method to the multiple bodies such as VLFS with the submerged plate to obtain the performance of VLFS.

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